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GRADE 5 • MODULE 6

Problem Solving with the Coordinate Plane

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Grade 5 • Module 6

Problem Solving with the Coordinate Plane

OVERVIEW

In this 40-day module, students develop a coordinate system for the first quadrant of the coordinate plane and use it to solve problems. Students use the familiar number line as an introduction to the idea of a coordinate and construct two perpendicular number lines to create a coordinate system on the plane. They see that just as points on the line can be located by their distance from 0, the plane's coordinate system can be used to locate and plot points using two coordinates. They then use the coordinate system to explore relationships between points, ordered pairs, patterns, lines and, more abstractly, the rules that generate them. This study culminates in an exploration of the coordinate plane in real world applications.

In Topic A, students come to realize that *any* line, regardless of orientation, can be made into a number line by first locating zero, choosing a unit length, and partitioning the length-unit into fractional lengths as desired. They are introduced to the concept of a coordinate as describing the distance of a point on the line from zero. As students construct these number lines in various orientations on a plane, they explore ways to describe the position of points *not* located on the lines. This discussion leads to the discovery that a second number line, perpendicular to the first, creates an efficient, precise way to describe the location of these points. Thus, points can be located using coordinate pairs, (a, b), by starting at the origin, travelling a distance of a units along the x-axis, and b units along a line parallel to the y-axis. Students describe given points using coordinate pairs as well as use given coordinate pairs to plot points (**5.G.1**). The topic concludes with an investigation of patterns in coordinate pairs along lines parallel to the axes, which leads to the discovery that these lines consist of the set of points whose distance from the x- or y-axis is constant.

Students move in to plotting points and using them to draw lines in the plane in Topic B (**5.G.1**). They investigate patterns relating the x- and y-coordinates of the points on the line and reason about the patterns in the ordered pairs, laying important groundwork for Grade 6 proportional reasoning**.** Topic B continues as students use given rules (e.g., multiply by 2, then add 3) to generate coordinate pairs, plot points, and investigate relationships. Patterns in the resultant coordinate pairs are analyzed, leading students to discover that such rules produce collinear sets of points. Students next generate two number patterns from two given rules, plot the points, and analyze the relationships within the sequences of the ordered pairs (**5.OA.3**). Patterns continue to be the focus as students analyze the effect on the steepness of the line when the second coordinate is produced through an addition rule as opposed to a multiplication rule (**5.OA.2**, **5.OA.3**). Students also create rules to generate number patterns, plot the points, connect those points with lines, and look for intersections.

Topic C finds students drawing figures in the coordinate plane by plotting points to create parallel, perpendicular, and intersecting lines. They reason about what points are needed to produce such lines and angles, and then investigate the resultant points and their relationships. Students also reason about the relationships among coordinate pairs that are symmetric about a line (**5.G.1**).

Problem solving in the coordinate plane is the focus of Topic D. Students draw symmetric figures using both angle size and distance from a given line of symmetry (**5.G.2**). Line graphs are also used to explore patterns and make predictions based on those patterns (**5.G.2, 5.OA.3**). To round out the topic, students use coordinate planes to solve real world problems.

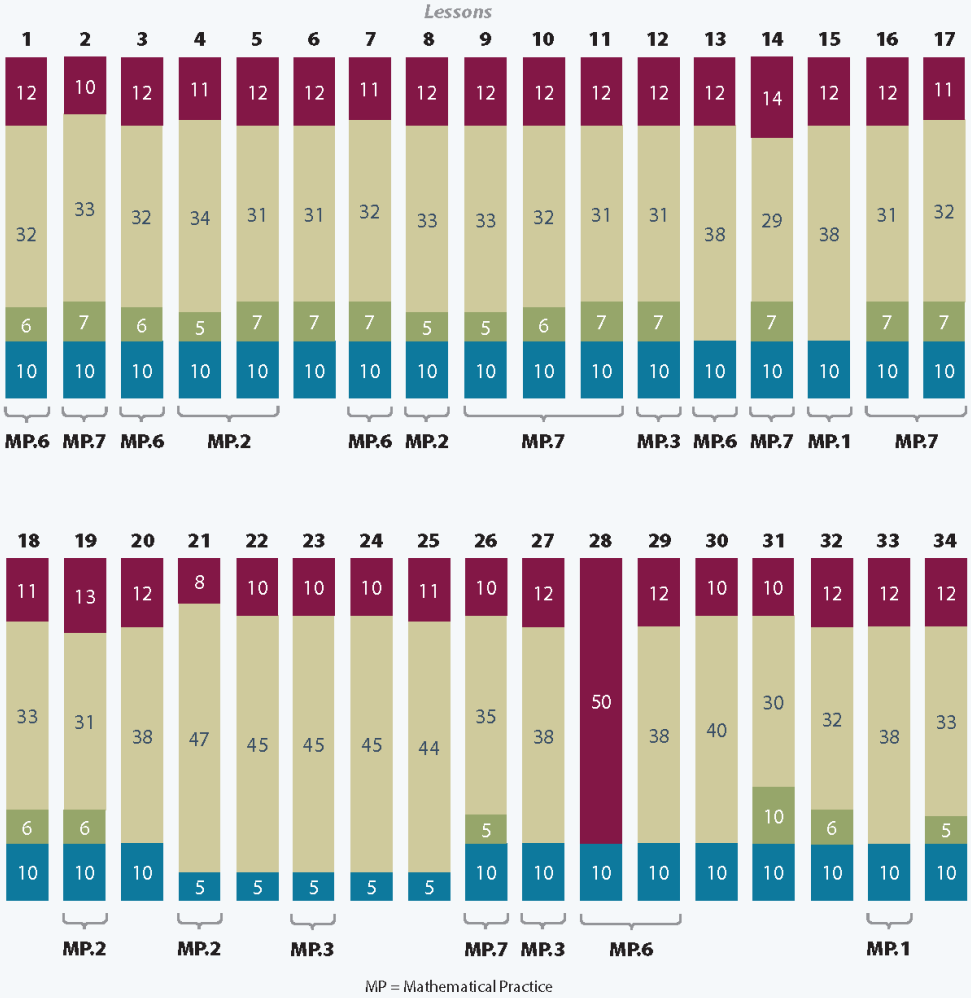
Topic E provides an opportunity for students to encounter complex, multi-step problems requiring the application of concepts and skills mastered throughout the Grade 5 curriculum. They use all four operations with both whole numbers and fractions in varied contexts. The problems in Topic E are designed to be non-routine, requiring students to persevere in order to solve them. While wrestling with complexity is an important part of Topic E, the true strength of this topic is derived from the time allocated for students to construct arguments and critique the reasoning of their classmates. After students have been given adequate time to ponder and solve the problems, two lessons are devoted to sharing approaches and solutions. Students will partner to justify their conclusions, communicate them to others, and respond to the arguments of their peers.

In this final topic of Module 6, and in fact, *A Story of Units*, students spend time producing a compendium of their learning. They not only reach back to recall learning from the very beginning of Grade 5, but they also expand their thinking by exploring such concepts as the Fibonacci sequence. Students solidify the year's learning by creating and playing games, exploring patterns as they reflect back on their elementary years. All materials for the games and activities are then housed for summer use in boxes created in the final two lessons of the year.

Distribution of Instructional Minutes

This diagram represents a suggested distribution of instructional minutes based on the emphasis of particular lesson components in different lessons throughout the module.

- Fluency Practice
- Concept Development
- Application Problems
- Student Debrief



MP = Mathematical Practice

Focus Grade Level Standards

Write and interpret numerical expressions.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product*.

Analyze patterns and relationships.

5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Foundational Standards

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two

rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.7c Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

c. Solve real world problems involving division of a unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students make sense of problems as they use tape diagrams and other models, persevering to solve complex, multi-step word problems. Students check their work and monitor their own progress, assessing their approach and its validity within the given context and altering their method when necessary.

MP.2 **Reason abstractly and quantitatively.** Students reason abstractly and quantitatively as they interpret the steepness and orientation of a line given by the points of a number pattern. Students attend to the meaning of the values in an ordered pair and reason about how they can be manipulated in order to create parallel, perpendicular, or intersecting lines.

MP.3 **Construct viable arguments and critique the reasoning of others.** As students construct a coordinate system on a plane, they generate explanations about the best place to create a second line of coordinates. They analyze lines and the coordinate pairs that comprise them, then draw conclusions and construct arguments about their positioning on the coordinate plane. Students also critique the reasoning of others and construct viable arguments as they analyze classmates' solutions to lengthy, multi-step word problems.

MP.6 **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They endeavor to use clear definitions in discussion with others and in their own reasoning. These students state the meaning of the symbols they choose, including using the equal sign, consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. The students calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

MP.7 **Look for and make use of structure.** Students identify and create patterns in coordinate pairs and make predictions about their effect on the lines that connect them. Students also recognize patterns in sets of coordinate pairs and use those patterns to explain why a line is parallel or perpendicular to an axis. They use operational rules to generate coordinate pairs and, conversely, generalize observed patterns within coordinate pairs as rules.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives		Days
5.G.1	A	**Coordinate Systems** Lesson 1: Construct a coordinate system on a line. Lesson 2: Construct a coordinate system on a plane. Lessons 3–4: Name points using coordinate pairs, and use the coordinate pairs to plot points. Lessons 5–6: Investigate patterns in vertical and horizontal lines, and interpret points on the plane as distances from the axes.	6
5.OA.2 **5.OA.3** **5.G.1**	B	**Patterns in the Coordinate Plane and Graphing Number Patterns from Rules** Lesson 7: Plot points, use them to draw lines in the plane, and describe patterns within the coordinate pairs. Lesson 8: Generate a number pattern from a given rule, and plot the points. Lesson 9: Generate two number patterns from given rules, plot the points, and analyze the patterns. Lesson 10: Compare the lines and patterns generated by addition rules and multiplication rules. Lesson 11: Analyze number patterns created from mixed operations. Lesson 12: Create a rule to generate a number pattern, and plot the points.	6
		Mid-Module Assessment: Topics A–B (assessment 1 day, return 1 day, remediation or further applications 1 day)	3
5.G.1 **5.G.2**	C	**Drawing Figures in the Coordinate Plane** Lesson 13: Construct parallel line segments on a rectangular grid. Lesson 14: Construct parallel line segments, and analyze relationships of the coordinate pairs. Lesson 15: Construct perpendicular line segments on a rectangular grid. Lesson 16: Construct perpendicular line segments, and analyze relationships of the coordinate pairs. Lesson 17: Draw symmetric figures using distance and angle measure from the line of symmetry.	5

Standards		Topics and Objectives	Days
5.OA.3 **5.G.2**	D	**Problem Solving in the Coordinate Plane** Lesson 18: Draw symmetric figures on the coordinate plane. Lesson 19: Plot data on line graphs and analyze trends. Lesson 20: Use coordinate systems to solve real world problems.	3
		End-of-Module Assessment: Topics A–D (assessment 1 day, return 1 day, remediation or further applications 1 day)	3
5.NF.2 5.NF.3 5.NF.6 5.NF.7c 5.MD.1 5.MD.5 5.G.2	E	**Multi-Step Word Problems** Lessons 21–25: Make sense of complex, multi-step problems and persevere in solving them. Share and critique peer solutions.	5
	F	**The Years in Review: A Reflection on *A Story of Units*** Lessons 26–27: Solidify writing and interpreting numerical expressions. Lesson 28: Solidify fluency with Grade 5 skills. Lessons 29–30: Solidify the vocabulary of geometry. Lesson 31: Explore the Fibonacci sequence. Lesson 32: Explore patterns in saving money. Lessons 33–34: Design and construct boxes to house materials for summer use.	9
Total Number of Instructional Days			**40**

Terminology

New or Recently Introduced Terms

- Axis (fixed reference line for the measurement of coordinates)
- Coordinate (number that identifies a point on a plane)
- Coordinate pair (two numbers that are used to identify a point on a plane; written (x, y) where x represents a distance from 0 on the x-axis and y represents a distance from 0 on the y-axis)
- Coordinate plane (plane spanned by the x-axis and y-axis in which the coordinates of a point are distances from the two perpendicular axes)
- Ordered pair (two quantities written in a given fixed order, usually written as (x, y))
- Origin (fixed point from which coordinates are measured; the point at which the x-axis and y-axis

intersect, labeled (0, 0) on the coordinate plane)
- Quadrant (any of the four equal areas created by dividing a plane by an x-axis and y-axis)

Familiar Terms and Symbols[1]

- Angle (union of two different rays sharing a common vertex)
- Angle measure (number of degrees in an angle)
- Degree (unit used to measure angles)
- Horizontal (parallel to the x-axis)
- Line (two-dimensional object that has no endpoints and continues on forever in a plane)
- Parallel (two lines in a plane that do not intersect)
- Perpendicular (two lines are *perpendicular* if they intersect, and any of the angles formed between the lines are 90-degree angles)
- Point (zero-dimensional figure that satisfies the location of an ordered pair)
- Rule (procedure or operation(s) that affects the value of an ordered pair)
- Vertical (parallel to the y-axis)

Suggested Tools and Representations

- Ruler
- Protractor
- Set square
- Tape diagrams

Scaffolds[2]

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*."

[1] These are terms and symbols students have seen previously.

[2] Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website, www.p12.nysed.gov/specialed/aim, for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.

Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	5.OA.2 5.OA.3 5.G.1
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	5.OA.2 5.OA.3 5.G.1 5.G.2